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ON FUZZY SUB IS-ALGEBRAS

Sundus Najah Jabir

Faculty of Education, Kufa, University, Iraq

ABSTRACT

In this paper we study sub IS-, algebra, fuzzy sub IS-, algebra, normal sub IS-algebra, fuzzy normal sub IS-algebra, fuzzy normal sub IS-algebra of fuzzy sub IS-algebra.

***KEYWORDS**: BCI-Algebras, Semigroup, IS-Algebra, Sub IS-Algebra, IS-Algebra Homomorphism, The Cartesian Product, Fuzzy Sub IS-Algebra, Normal Sub IS-Algebra*

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1. INTRODUCTION

In 1996, K. Iseki introduced the notion of BCK/BCI- algebras. For the general development of BCK/BCI- algebras [6], In [2]introduced a new class of algebras related to BCI- algebras and semi groups called a BCI- semi group. In this paper we study a new type of fuzzy sub IS-algebra are normal sub IS-algebra, fuzzy normal sub IS-algebra and fuzzy normal sub IS-algebra of fuzzy sub IS-algebra.

2. PRELIMINARY

We review some definitions that will be useful in our results.

***Definition* 2.1:** A Semi group is an ordered pair $(G,\cdot)$,where G is a non-empty set and " ." is an associative binary operation on G. [3]

***Definition* 2.2** A BCI- algebra is triple (G, *, 0) where G is a non-empty set "*" is binary operation on G, $0 \in G$ is an element such that the following axioms are satisfied for all $s, t, r \in G$:

- $((s * t) * (s * r)) * (r * t) = 0$,
- $(s * (s * t)) * t = 0$,
- $s * s = 0$,
- $s * t = 0$ and $t * s = 0$ implies $= t$

If $0 * s = 0$ for all $s \in G$ then G is called BCK-algebra. [1]

***Definition* 2.3:** An IS-algebra is a non-empty set with two binary operation "*" and "." and constant 0 satisfying the axioms:

- (G,∗, 0) is a BCI-algebra.
- (G, .)is a Semi group,
- s.(t ∗r) = (s.t) ∗(s.r) and (s ∗t).r = (s.r) ∗(t.r), for all s, t, r ∈ *G*. [6]

***Example* 2.4:** let G={0,a,b,c} define "*" operation and multiplication "." by the following tables:

*	0	a	b	c
0	0	0	b	b
a	a	0	c	b
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then by routine calculations we can see that G is an IS-algebra.[6]

***Definition* 2.5:** Let G and Y be IS-algebra a mapping $f : G \to Y$ is called an IS-algebra homomorphism (briefly homomorphism) if $f(x * y) = f(x) * f(y)$ and $f(xy) = f(x)f(y)$ for all $x, y \in G$.

Let $f : G \to Y$ IS-algebra homomorphism. Then the set $\{x \in G : f(x) = 0\}$ is called the kernel of *f*, and denote by *Ker f*. Moreover, the set $\{f(x) \in Y : x \in G\}$ is called the image of f and denote by Im f. [4]

***Definition* 2.6:** Let λ and μ be the fuzzy subsets in a set G, the Cartesian product

$\lambda \times \mu$: G × G $\longrightarrow$[0, 1] is defined by$(\lambda \times \mu)(x, y) = \min\{\lambda(x), \mu(y)\}$

for all $x, y \in G$. [9]

***Definition* 2.7:** Let G be a non-empty set a fuzzy subset « of G is a function μ : G→[0, 1]. [10]

***Definition* 2.8:** Let « and ¬ be a fuzzy sets on G. Define the fuzzy set $« \cap ¬$ as follows: $(« \cap ¬)(x) = \min\{«(x), ¬(x)\}$ for all $x \in G$.[5]

***Definition* 2.9:** Let « and ¬ be a fuzzy sets on G . Define the fuzzy set $« \cup ¬$ as follows:

$(« \cup ¬)(x) = \max\{«(x), ¬(x)\}$ for all $x \in G$.[5]

3. MAIN RESULTS

In this section, we find some results about fuzzy sub IS-algebra, normal sub IS-algebra, fuzzy normal sub IS-algebra and fuzzy normal sub IS-algebra of fuzzy sub IS-algebra.

***Definition* 3.1:** A fuzzy set « defined on G is called a fuzzy sub IS-algebra of G if it satisfies the following conditions:

1) $«(x_1 * x_2) \geq \min\{«(x_1), «(x_2)\}$,
2) $«(x_1 x_2) \geq \min\{«(x_1), «(x_2)\}$ $\forall\ x_1, x_2 \in G$

Proposition 3.2: Let « and ¬ be fuzzy IS-algebra of G. Then $« \cap ¬$ is a fuzzy IS-algebra of G .

Proof: Let « and ¬ are the fuzzy sub IS-algebra and let $x, y \in « \bigcap ¬$ then

$$\begin{aligned}(« \cap ¬)(xy) &= \min\{«(xy), ¬((xy)\} \\ &\geq \min\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by \text{ hypothesis}] \\ &= \min\{\min\{«(x), ¬(x)\}, \min\{«(y), ¬(y)\}\} \\ &= \min\{(« \cap ¬)(x), (« \cap ¬)(y)\}.\end{aligned}$$

so,

$$\begin{aligned}(« \cap ¬)(x * y) &= \min\{«(x * y), ¬((x * y)\} \\ &\geq \min\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by \text{ hypothesis }] \\ &= \min\{\min\{«(x), ¬(x)\}, \min\{«(y), ¬(y)\}\} \\ &= \min\{(« \cap ¬)(x), (« \cap ¬)(y)\}.\end{aligned}$$

Hence $« \cap ¬$ is a fuzzy sub IS-algebra.

Proposition 3.3: Let « and ¬ are fuzzy sub IS-algebra of G then $« \bigcup ¬$ is a fuzzy sub IS-algebra of G if $« \subseteq ¬ \ \ or \ \ ¬ \subseteq «$.

Proof: Let « and ¬ are the fuzzy sub IS-algebra, and let $x, y \in « \bigcup ¬$ then

$$\begin{aligned}(« \cup ¬)(xy) &= \max\{«(xy), ¬((xy)\} \\ &\geq \max\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by\ hypthoses] \\ &= \min\{\max\{«(x), ¬(x)\}, \max\{«(y), ¬(y)\}\} \quad [« \subseteq ¬ \ or \ ¬ \subseteq «] \\ &= \min\{(« \cup ¬)(x), (« \cup ¬)(y)\}.\end{aligned}$$

so,

$$\begin{aligned}(« \cup ¬)(x * y) &= \max\{«(x * y), ¬((x * y)\} \\ &\geq \max\{\min\{«(x), «(y)\}, \min\{¬(x), ¬(y)\}\} \quad [by\ hypthoses] \\ &= \min\{\max\{«(x), ¬(x)\}, \max\{«(y), ¬(y)\}\} \quad [« \subseteq ¬ \ or \ ¬ \subseteq «] \\ &= \min\{(« \cup ¬)(x), (« \cup ¬)(y)\}.\end{aligned}$$

Hence $\mu \cup v$ is a fuzzy sub IS-algebra.

Proposition 3.4: Let G be a IS-algebra and let μ, ν, be a fuzzy sub IS-algebra then $\mu \times$ ν is a fuzzy sub IS-algebra of $G \times G$.

Proof: Let « and ¬ are fuzzy IS-algebra ∋ $(x_1, y_1), (x_2, y_2) \in G \times G$ then

$$\begin{aligned}(« \times ¬)((x_1, y_1).(x_2, y_2)) &= (« \times ¬)((x_1.x_2, y_1.y_2)) \\ &= \min\{«(x_1.x_2), ¬(y_1.y_2)\} \\ &\geq \min\{\min\{«(x_1), «(x_2)\}, \min\{¬(y_1), ¬(y_2)\}\} \\ &= \min\{\min\{«(x_1), ¬(y_1)\}, \min\{«(x_2), ¬(y_2)\}\} \\ &= \min\{(« \times ¬)(x_1, y_1), (« \times ¬)(x_2, y_2)\}\end{aligned}$$

$$\begin{aligned}(\text{«}\times\neg)((x_1,y_1)*(x_2,y_2)) &= (\text{«}\times\neg)((x_1*x_2,y_1*y_2))\\ &=\min\{\text{«}(x_1*x_2),\neg(y_1*y_2)\}\\ &\geq \min\{\min\{\text{«}(x_1),\text{«}(x_2)\},\min\{\neg(y_1),\neg(y_2)\}\}\\ &=\min\{\min\{\text{«}(x_1),\neg(y_1)\},\min\{\text{«}(x_2),\neg(y_2)\}\}\\ &=\min\{(\text{«}\times\neg)(x_1,y_1),(\text{«}\times\neg)(x_2,y_2)\}\end{aligned}$$

Hence $\text{«}\times\neg$ is a fuzzy sub IS-algebra.

***Definition* 3.5:** A fuzzy sub IS-algebra « of G is said to be normal fuzzy sub IS-algebra if there exists $x \in G$ such that $\text{«}(x) = 1$.

Remark 3.6: A fuzzy sub IS-algebra μ of G is said to be normal fuzzy sub IS-algebra if and only if $\mu(0) = 1$.

Proof:

Let μ be a normal fuzzy sub IS-algebra of G then

there exists $x \in G$ *such that* $\mu(x) = 1$

since $\mu(0) \geq \mu(x) \quad \forall\, x \in G$

so $\mu(0) \geq 1$ then $\mu(0) = 1$.

Conversely, it is clear.

Proposition 3.7: Let μ and v are normal fuzzy sub IS-algebra of G then $\mu \cap v$ be a normal fuzzy sub IS-algebra of G.

Proof:

Let μ and v are normal fuzzy sub IS-algebra of G then

$\mu \cap v$ is a fuzzy sub IS-algebra of G [by Proposition (3.2)]

also $\mu(0) = 1$ and $v(0) = 1$ so

$$(\text{«}\cap\neg)(0) = \min\{\text{«}(0),\neg(0)\} = 1$$

therefore $(\text{«}\cap\neg)$ is a normal fuzzy sub IS-algebra.

Proposition 3.8: Let « and $\neg$ are normal fuzzy sub IS-algebra of G then $\text{«}\cup\neg$ be a normal fuzzy sub IS-algebra of G if $\text{«}\subseteq\neg$ *or* $\neg\subseteq\text{«}$.

Proof:

Let μ and v are normal fuzzy sub IS-algebra of G such that $\text{«}\subseteq\neg$ *or* $\neg\subseteq\text{«}$ then

$\text{«}\cup v$ is a fuzzy sub IS-algebra of G [by Proposition (3.3)]

also $\mu(0) = 1$ and $v(0) = 1$ so

$(\mu \cup \nu)(0) = \max\{\mu(0), \nu(0)\} = 1$

therefore $\mu \cup \nu$ is a normal fuzzy sub IS-algebra.

Proposition 3.9: Let μ and v be a normal fuzzy sub IS-algebra then $\mu \times \nu$ is a normal fuzzy sub IS-algebra.

Proof:

Let μ and v are normal fuzzy sub IS-algebra of G then,

since μ and v are fuzzy sub IS-algebra

so [by Proposition (3.4)] $\mu \times \nu$ is a fuzzy sub IS-algebra

Now,

$(\mu \times \nu)(0,0) = \min\{\mu(0), \nu(0)\} = \min\{1,1\} = 1$

[since μ, v are normal fuzzy sub IS-algebra]

Hence $\mu \times \nu$ is normal fuzzy sub IS-algebra.

Definition 3.10: Let G be a IS-algebra and μ a fuzzy set on X . Then μ is called a fuzzy normal sub IS-algebra of G if it satisfies the following conditions:

1) μ is a fuzzy sub IS-algebra of G .
2) $\mu(x * y) = \mu(y * x) \quad \forall x, y \in G \setminus \{0\}$.
3) $\mu(xy) = \mu(yx) \quad \forall x, y \in G.$

Proposition 3.11: Let μ and v are fuzzy normal sub IS-algebra of G then $\mu \cap \nu$ be a fuzzy normal sub IS-algebra.

Proof:

Let μ and v are fuzzy normal sub IS-algebra of G,

then $\mu \cap \nu$ is a fuzzy sub IS-algebra of G [by Proposition (3.2)]

Now,

$$
\begin{aligned}
(\mu \cap \nu)(xy) &= \min\{\mu(xy), \nu(xy)\} \\
&= \min\{\mu(yx), \nu(yx)\} \ [\mu, \nu \ \textit{are fuzzy normal sub} \mathrm{IS\text{-}algebra}] \ \text{so,} \\
&= (\mu \cap \nu)(yx) \ , \quad \forall \ x, y \in G.
\end{aligned}
$$

$$
\begin{aligned}
(\mu \cap \nu)(x * y) &= \min\{\mu(x * y), \nu(x * y)\} \\
&= \min\{\mu(y * x), \nu(y * x)\} [\mu, \nu \ \textit{are fuzzy normal sub} \ \mathrm{IS\text{-}algebra}] \\
&= (\mu \cap \nu)(y * x) \quad \forall \ x, y \in G \setminus \{0\}.
\end{aligned}
$$

therefore $\mu \cap \nu$ is a fuzzy normal sub IS-algebra.

Proposition 3.12: Let μ and v are fuzzy normal sub IS-algebra of G . Then $« \cup ¬$ be a fuzzy normal sub IS-algebra if $« \subseteq ¬ \ or\ ¬ \subseteq «$.

Proof:

Suppose that μ and v are fuzzy normal sub IS-algebra

then μ and v are fuzzy sub IS-algebra then

$« \cup ¬$ be a fuzzy sub IS-algebra [by Proposition (3.3)]

Now,

$$\begin{aligned}(« \cup ¬)(xy) &= \max\{«(xy), ¬(xy)\} \\ &= \max\{«(yx), ¬(yx)\} \qquad [by\ hypothesis\ ] \\ &= (« \cup ¬)(yx) \qquad \forall\ x, y \in G.\end{aligned}$$

so,

$$\begin{aligned}(« \cup ¬)(x * y) &= \max\{«(x * y), ¬(x * y)\} \\ &= \max\{«(y * x), ¬(y * x)\} \qquad [by\ hypothesis\ ] \\ &= (« \cup ¬)(y * x) \qquad \forall\ x, y \in G \setminus \{0\}.\end{aligned}$$

Hence $« \cup ¬$ is a fuzzy normal sub IS-algebra.

Proposition 3.13: Let ª and « are fuzzy normal sub IS-algebra of G then $ª \times «$ is a fuzzy normal sub IS-algebra of $G \times G$.

Proof:

Let λ and μ be a fuzzy normal sub IS-algebra of G and let

$(x_1,x_2), (y_1,y_2) \in G \times G$ *where* $x_1,x_2, y_1,y_2 \in G \ni x = (x_1,x_2), y = (y_1,y_2)$

then λ and μ be a fuzzy sub IS-algebra of G so

$\lambda \times \mu$ is a fuzzy sub IS-algebra [by Proposition (3.4)]

now,

$$\begin{aligned}(ª \times «)(xy) &= (ª \times «)((x_1, x_2) \cdot (y_1, y_2)) \\ &= (ª \times «)(x_1y_1, x_2y_2) \\ &= \min\{ª(x_1y_1), «(x_2y_2)\} \\ &= \min\{ª(y_1x_1), «(y_2x_2)\} \quad [ª, «\ are\ fuzzy\ normal\ sub\text{IS-algebra}] \\ &= (ª \times «)((y_1, y_2) \cdot (x_1, x_2)) \\ &= (ª \times «)(yx)\end{aligned}$$

and so,

$let\ \ (x_1,x_2),\ (y_1,y_2)\in G\times G\ \ where\ \ x_1,x_2,y_1,y_2\in G\setminus\{0\}$
$such\ that\ \ \ x=(x_1,x_2)\ ,\ y=(y_1,y_2)\in G\times G$

$$
\begin{aligned}
(ª\times«)(x*y) &= (ª\times«)((x_1,x_2)*(y_1,y_2))\\
&= (ª\times«)(x_1*y_1,x_2*y_2)\\
&= \min\{ª(x_1*y_1),«(x_2*y_2)\}\\
&= \min\{ª(y_1*x_1),«(y_2*x_2)\}\ [ª,«\ are\ fuzzy\ normal\ sub\text{IS-algebras}\ ]\\
&= (ª\times«)((y_1,y_2)*(x_1,x_2))\\
&= (ª\times«)(y*x)
\end{aligned}
$$

therefore $ª\times«$ is a fuzzy normal sub IS-algebra.

Proposition 3.14: Let G be a IS-algebra and $«$, $ª$ be two fuzzy sets in G such that $«\times ª$ is a fuzzy sub IS-algebra of $G\times G$. Then:

1) either $«(x)\le«(0)\ \ or\ \ ª(x)\le ª(0)\ \ for\ all\ \ x\in G$.
2) If $«(x)\le«(0)\ \ for\ all\ \ x\in \mathrm{X}$ then either $«(x)\le ª(0)\ \ or\ \ ª(x)\le ª(0)$.
3) If $ª(x)\le ª(0)\ \ for\ all\ \ x\in \mathrm{X}$ then either $«(x)\le«(0)\ \ or\ \ ª(x)\le«(0)$.
4) either $«$ or $ª$ is a fuzzy sub IS-algebra of G .

Proposition 3.15: Let $«\times ª$ be a fuzzy normal sub IS-algebra of G then either $ª$ or $«$ is a fuzzy normal sub IS-algebra of G .

Proof:

Let $«\times ª$ be a fuzzy normal sub IS-algebra of G

so $«\times ª$ be a fuzzy sub IS-algebra of G

then by use Proposition (3.14), either $ª$ or $«$ is a fuzzy sub IS-algebra of G

if $ª$ be a fuzzy sub IS-algebra of G

so [by (3.14)] $ª(x)\le«(0)$

to prove $ª$ is a normal

let $x_1,x_2\in \mathrm{X}$ then

$$
\begin{aligned}
ª(x_1x_2) &= \min\{«(0), ª(x_1x_2)\} \\
&= («\times ª)(0, x_1x_2) \\
&= («\times ª)((0, x_1)\cdot(0, x_2)) \\
&= («\times ª)((0, x_2)\cdot(0, x_1)) \\
&= («\times ª)(0, x_2x_1) \\
&= \min\{«(0), ª(x_2x_1)\} \\
&= ª(x_2x_1)
\end{aligned}
$$

Now, let $x_1, x_2 \in G/\{0\}$

$$
\begin{aligned}
ª(x_1 * x_2) &= \min\{«(0), ª(x_1 * x_2)\} \\
&= («\times ª)(0, x_1 * x_2) \\
&= («\times ª)((0, x_1) * (0, x_2)) \\
&= («\times ª)((0, x_2) * (0, x_1)) \quad [«\times ª \textit{ is a fuzzy normal sub}\text{IS-algebra}\ ] \\
&= («\times ª)(0, x_2 * x_1) \\
&= \min\{«(0), ª(x_2 * x_1)\} \\
&= ª(x_2 * x_1) \qquad \forall\ x_1, x_2 \in G\setminus\{0\}\ .
\end{aligned}
$$

Hence $ª$ is a fuzzy normal sub IS-algebra.

In similar way. if $«\times ª$ is a fuzzy normal sub IS-algebra and $«$ is a fuzzy sub IS-algebra.

We can prove that $«$ is a fuzzy normal sub IS-algebra.

***Definition* 3.16:** Let G be a IS-algebra, $«$ and $¬$ are fuzzy sub IS-algebra of G such that $« \subseteq ¬$ then $«$ is called fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$ if :

$$
\begin{aligned}
&(1) \quad «(y * x) \geq \min\{«(x * y), ¬(y)\} \\
&(2) \quad «(yx) \geq \min\{«(xy), ¬(y)\} \quad , \ \forall\ x, y \in X.
\end{aligned}
$$

***Proposition 3.17*:** Let G be a IS-algebra and let $«$ and $ª$ be fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$. Then $« \cap ª$ is a fuzzy normal sub IS-algebra of $¬$.

Proof:

Let $«$ and $ª$ are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$.

Then $« \cap ª$ is a fuzzy sub IS-algebra [by Proposition (3.2)]

Now, let $x, y \in X$, since

$«(y * x) \geq \min\{«(x * y), ¬(y)\}$, $ª(y * x) \geq \min\{ª(x * y), ¬(y)\}$ and

$«(yx) \geq \min\{«(xy), ¬(y)\}$, $ª(yx) \geq \min\{ª(xy), ¬(y)\}$

therefore

$$
\begin{aligned}
1)\quad (« \cap ª)(yx) &= \min\{«(yx), ª(yx)\} \\
&\geq \min\{\min\{«(xy), ¬(y)\}, \min\{ª(xy), ¬(y)\}\} \\
&= \min\{\min\{«(xy), ª(xy)\}, \min\{¬(y), ¬(y)\}\} \\
&= \min\{(« \cap ª)(xy), ¬(y)\}
\end{aligned}
$$

and,

$$
\begin{aligned}
2)\quad (« \cap ª)(y*x) &= \min\{«(y*x), ª(y*x)\} \\
&\geq \min\{\min\{«(x*y), ¬(y)\}, \min\{ª(x*y), ¬(y)\}\} \\
&= \min\{\min\{«(x*y), ª(x*y)\}, \min\{¬(y), ¬(y)\}\} \\
&= \min\{(« \cap ª)(x*y), ¬(y)\}
\end{aligned}
$$

Hence $« \cap ª$ is a fuzzy normal sub IS-algebra of $¬$.

***Proposition 3.18*:** Let X be a IS-algebra and let $«$ and $ª$ are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$ then $« \cap ª$ is a fuzzy normal sub IS-algebra of $¬$ if $« \subseteq ª$ or $ª \subseteq «$.

Proof:

Let $«$ and $ª$ are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$.

$« \cup ¬$ is a fuzzy sub IS-algebra[by Proposition (3.3)]

Now, let $x, y \in G$ then

$$
\begin{aligned}
1)\quad (« \cup ª)(yx) &= \max\{«(yx), ª(yx)\} \\
&\geq \max\{\min\{«(xy), ¬(y)\}, \min\{ª(xy), ¬(y)\}\} \\
&= \min\{\max\{«(xy), ª(xy)\}, \max\{¬(y), ¬(y)\}[\text{since}\ « \subseteq ª\ or\ ª \subseteq «] \\
&= \min\{(« \cup ª)(xy), ¬(y)\}
\end{aligned}
$$

and so,

$$
\begin{aligned}
2)\quad (« \cup ª)(y*x) &= \max\{«(y*x), ª(y*x)\} \\
&\geq \max\{\min\{«(x*y), ¬(y)\}, \min\{ª(x*y), ¬(y)\}\} \\
&= \min\{\max\{«(x*y), ª(x*y)\}, \max\{¬(y), ¬(y)\}[« \subseteq ª\ or\ ª \subseteq «] \\
&= \min\{(« \cup ª)(x*y), ¬(y)\}
\end{aligned}
$$

Hence $« \cup ª$ is a fuzzy normal sub IS-algebra of $¬$.

***Proposition 3.19*:** If $«$ and $ª$ are fuzzy normal sub IS-algebra of fuzzy sub IS-algebra $¬$ then $« \times ª$ is a fuzzy normal sub IS-algebra of $¬ \times ¬$.

Proof:

Let $«$ and $ª$ are fuzzy normal sub IS-algebra of $¬$.

let $(x_1,x_2), (y_1,y_2) \in G \times G$ *such that* $x = (x_1,x_2), y = (y_1,y_2)$

so ¬ , « , ª are fuzzy sub IS-algebra of G,

then $\neg \times \neg$ is a fuzzy sub IS-algebra [by Proposition (3.9)]

then « × ª is a fuzzy sub IS-algebra of $G \times$ G [by Proposition (3.9)] .

Now, to prove « × ª is a fuzzy normal sub IS-algebra of $\neg \times \neg$

$$
\begin{aligned}
(« \times ª)(yx) &= (« \times ª)((y_1,y_2)(x_1,x_2)) \\
&= (« \times ª)(y_1x_1, y_2x_2) \\
&= \min\{«(y_1x_1), ª(y_2x_2)\} \\
&\geq \min\{\min\{«(x_1y_1), \neg(y_1)\}, \min\{ª(x_2y_2), \neg(y_2)\}\} \\
&= \min\{\min\{«(x_1y_1), ª(x_2y_2)\}, \min\{\neg(y_1), \neg(y_2)\}\} \\
&= \min\{(« \times ª)((x_1,x_2)(y_1,y_2)), \neg \times \neg(y_1,y_2)\} \\
&= \min\{(« \times ª)(xy), \neg \times \neg(y)\}
\end{aligned}
$$

and so,

$$
\begin{aligned}
(« \times ª)(y*x) &= (« \times ª)((y_1,y_2)*(x_1,x_2)) \\
&= (« \times ª)(y_1*x_1, y_2*x_2) \\
&= \min\{«(y_1*x_1), ª(y_2*x_2)\} \\
&\geq \min\{\min\{«(x_1*y_1), \neg(y_1)\}, \min\{ª(x_2*y_2), \neg(y_2)\}\} \\
&= \min\{\min\{«(x_1*y_1), ª(x_2*y_2)\}, \min\{\neg(y_1), \neg(y_2)\}\} \\
&= \min\{(« \times ª)((x_1,x_2)*(y_1,y_2)), \neg \times \neg(y_1,y_2)\} \\
&= \min\{(« \times ª)(x*y), \neg \times \neg(y)\}
\end{aligned}
$$

Hence « × ª is a fuzzy normal sub IS-algebra of $\neg \times \neg$.

***Proposition 3.20*:** Let $f : G \to \mathrm{Y}$ be a homomorphism if « is a normal fuzzy sub IS-algebra of Y then $«^f$ is a normal fuzzy sub IS-algebra of G.

***Proposition 3.21*:** Let $f : G \to \mathrm{Y}$ be a homomorphism if « is a fuzzy normal sub IS-algebra of a fuzzy sub IS-algebra ¬ . Then $«^f$ is a fuzzy normal sub IS-algebra of $\neg^f$.

Proof:

Let « is a fuzzy normal sub IS-algebra of ¬ . Then

$$«^f(x) = «(f(x)) \leq \neg(f(x)) = \neg^f(x) \qquad [\sin ce\ « \subseteq \neg].$$

and $«^f$ is a fuzzy sub IS-algebra[by Proposition (3.20)]

$\neg^f$ is a fuzzy sub IS-algebra

Now, to prove $«^f$ is a fuzzy normal sub IS-algebra of $\neg^f$ thus

$$\begin{aligned}
«^{f}(yx) &= «(f(yx)) \\
&= «(f(y)f(x)) \\
&\geq \min\{«(f(x)f(y)), \neg(f(y))\} \\
&= \min\{«(f(xy)), \neg^{f}(y)\} \\
&= \min\{«^{f}(xy), \neg^{f}(y)\}
\end{aligned}$$

so,

$$\begin{aligned}
«^{f}(y*x) &= «(f(y*x)) \\
&= «(f(y)*f(x)) \\
&\geq \min\{«(f(x)*f(y)), \neg(f(y))\} \\
&= \min\{«(f(x*y)), \neg^{f}(y)\} \\
&= \min\{«^{f}(x*y), \neg^{f}(y)\}
\end{aligned}$$

Hence $«^{f}$ is a fuzzy normal sub IS-algebra of $\neg^{f}$.

***Proposition 3.22*: ** Let $f : G \to \mathrm{Y}$ be epimorphism if $«^{f}$ is a normal fuzzy sub IS-algebra of G then $«$ is a normal fuzzy sub IS-algebra of Y.

***Proposition 3.23*: ** Let $f : G \to \mathrm{Y}$ epimorphism if $«^{f}$ is a fuzzy normal sub IS-algebra of $\neg^{f}$. Then $«$ is a fuzzy normal sub IS-algebra of V .

Proof:

Let $«^{f}$ is a fuzzy normal sub IS-algebra of $\neg^{f}$ then

since f is an epimorphism if $x \in \mathrm{Y} \ \ \exists\, a \in \mathrm{X}$ *such that* $f(a) = x$

$$«(x) = «(f(a)) = «^{f}(a) \leq \neg^{f}(a) = \neg(f(a)) = \neg(x) \quad so \quad «(x) \subseteq \neg(x) \ \ \forall x \in \mathrm{Y} \ .$$

and $«$ is a fuzzy sub IS-algebra[by Proposition (3.22)]

Now, let $x, y \in \mathrm{Y} \ \ \exists\, a, b \in G$ *such that* $f(a) = x$, $f(b) = y$ then

$$\begin{aligned}
«(yx) &= «(f(b)f(a)) = «(f(ba)) = «^{f}(ba) \\
&\geq \min\{«^{f}(ab), \neg^{f}(b)\} \\
&= \min\{«(f(ab)), \neg(f(b))\} \\
&= \min\{«(f(a)f(b)), \neg(y)\} \\
&= \min\{«(xy), \neg(y)\}
\end{aligned}$$

and so,

$$
\begin{aligned}
«(y*x) &= «(f(b)*f(a)) \\
&= «(f(b*a)) \\
&= «^{f}(b*a) \\
&\geq \min\{«^{f}(a*b), ¬^{f}(b)\} \\
&= \min\{«(f(a*b)), ¬(f(b))\} \\
&= \min\{«(f(a)*f(b)), ¬(y)\} \\
&= \min\{«(x*y), ¬(y)\}
\end{aligned}
$$

Hence « is a fuzzy normal sub IS-algebra of ¬ .

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